SPC: Control Chart Fundamentals and Applications

9th Annual ASQ Illiana Quality Conference

Dan Mateja

January 22, 2016
Background

• 21 years in the steel industry
  – Quality
  – Process Technology
• BS and MS in Metallurgical Engineering and Materials Science
• MS in Quality Assurance
• ASQ CQM/OE
• Certified Black Belt through Whirlpool Operational Excellence
• ASQ Member since 2009
  – Illiana Chief ASQ Exam Proctor
  – Illiana Section Chair
Agenda

• 7 Quality Tools for Process Improvement

• Control Chart History and Overview

• Variable Control Charts
  – Average (X-bar) – Range (R)
  – Other Variable Control Charts

• Attribute Control Charts
  – P-Chart
  – Other Attribute Control Charts
Seven Quality Tools

• Cause-and-Effect Diagram – Fishbone Chart
• Check Sheet
• **Control Charts**
• Histogram
• Pareto Chart
• Scatter Diagram
• Stratification
Control Chart History and Overview

• Invented by Walter A. Shewhart in the 1920’s
  – Improve reliability of Bell Lab telephone transmission lines
  – Reduce frequency of failures and repairs
  – Recognized common and special cause variation
  – Need to bring process in control to predict the future and make a process economically

• Mid 1920’s - W. Edwards Deming recognized significance
  – Statistical consultant to Post-World War II Japan
  – Used control chart in Japanese manufacturing industry in 1950’s and 1960’s
Control Chart History and Overview

- Basic Definition: Control Charts are tools used to determine if a process is in a state of control.
Control Chart History and Overview

- Control Charts used for several purposes
  - **Monitor** process variables and parameters. Assess the stability of parameter and to “flag” when a process goes out of control. **SPC.**
  
  - **Validate** the effect of changes on process parameters. Assess the effectiveness of a change.
  
  - Useful in **improvement** activities. Understand the relationship between process variables and parameters.
  
  - Useful to understand the stability and **variation in critical process variables** indentified in trials/experiments (DOE’s). Control charts used in follow-up studies.
Control Chart History and Overview

• Control Charts Basics

  – Graph used to study how a process changes over time
  – Data plotted in time order
  – Central line for average, upper line for upper control limit and a lower line for lower control limit
  – Line and limits determined from historical data
  – Conclusions about whether the process variation is consistent (in control) or is unpredictable (unstable, out of control, affected by special causes of variation)
Control Chart History and Overview

• Control Chart Purpose for SPC

  – Recognize the extent of variation currently exists
    • Do not react to random variation

  – Study the process to identify sources of variation
    • Act to eliminate or reduce variation sources
      – Special causes
      – Common causes
Control Chart History and Overview

- **Types of Control Charts**
  - **Variable**
    - Continuous data
    - Control charts used in pairs
      - One chart (typically the top) monitors the average or centering of the process distribution
      - The other chart (typically the bottom) monitors the range of the distribution
  - **Attribute**
    - Non-continuous, Go-No Go, Pass-Fail
    - A single control chart
Control Chart History and Overview

Control Chart Decision Tree

Choose Appropriate Control Chart

Attribute Data
Counted & plotted as discrete events

Defect Data
- Constant sample size
  - c Chart

Defective Data
- Variable sample size
  - u Chart
- Constant sample size ≥ 50
  - np Chart
- Variable sample size ≥ 50
  - p Chart

Continuous Data
Measured & plotted on a continuous scale

Sample size = 1
- I and MR

Sample is large, usually ≥ 10
- X-Bar and s

Sample is small, usually 3 to 5
- X-Bar and R
Variable Control Charts – X-Bar-R

Quality Tools

Control Charts

Description
This template illustrates a Statistical Process Control (SPC) chart. A detailed discussion of SPC charts can be found at www.ASQ.org

Learn About Statistical Process Control

Instructions

- Select the correct subgroup size. When in doubt, select a subgroup size of one. Partial subgroups are not displayed.
  - One
  - Two
  - Three
  - Four
  - Five
  - Six
- Enter up to 200 data points in the cells provided. Do not enter values in the subgroup column. These cells update automatically to show the subgroup in which the data point is included.
- Identify any out of control conditions. Four tests are performed. Use the legend to identify the points corresponding to a particular test.
- If a test looks for a proportion of points, only the offending point will be identified. For example, if eight points in a row are on one side of the centerline only the eighth point will be identified.

Learn More

To learn more about other quality tools, visit the ASQ Learn About Quality web site.

Learn About Quality

X-Bar Chart - Pin Diameter

Range Chart - Pin Diameters

- Three Sigma Limit
- Two Sigma Limit
- One Sigma Limit
- Average

A single point outside the control limits
Two of three pts outside the two sigma limit
Four of Five pts outside the one sigma limit
Eight in a row on the same side of centerline
Variable Control Charts – X-Bar-R

<table>
<thead>
<tr>
<th>Sub Group</th>
<th>Data</th>
<th>Sub Group</th>
<th>Data</th>
<th>Sub Group</th>
<th>Data</th>
<th>Sub Group</th>
<th>Data</th>
<th>Sub Group</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.249</td>
<td>6</td>
<td>0.25</td>
<td>11</td>
<td></td>
<td>1</td>
<td>0.251</td>
<td>16</td>
<td>0.249</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>6</td>
<td>0.25</td>
<td>11</td>
<td>0.25</td>
<td>11</td>
<td>0.251</td>
<td>16</td>
<td>0.249</td>
</tr>
<tr>
<td>1</td>
<td>0.251</td>
<td>6</td>
<td>0.251</td>
<td>11</td>
<td>0.251</td>
<td>11</td>
<td>0.251</td>
<td>16</td>
<td>0.249</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>6</td>
<td>0.25</td>
<td>11</td>
<td>0.25</td>
<td>11</td>
<td>0.25</td>
<td>16</td>
<td>0.249</td>
</tr>
<tr>
<td>1</td>
<td>0.25</td>
<td>6</td>
<td>0.251</td>
<td>11</td>
<td>0.25</td>
<td>11</td>
<td>0.251</td>
<td>16</td>
<td>0.249</td>
</tr>
<tr>
<td>2</td>
<td>0.251</td>
<td>7</td>
<td>0.247</td>
<td>12</td>
<td>0.251</td>
<td>17</td>
<td>0.249</td>
<td>21</td>
<td>0.248</td>
</tr>
<tr>
<td>2</td>
<td>0.251</td>
<td>7</td>
<td>0.251</td>
<td>12</td>
<td>0.251</td>
<td>17</td>
<td>0.251</td>
<td>21</td>
<td>0.248</td>
</tr>
<tr>
<td>2</td>
<td>0.251</td>
<td>7</td>
<td>0.251</td>
<td>12</td>
<td>0.251</td>
<td>17</td>
<td>0.251</td>
<td>21</td>
<td>0.248</td>
</tr>
<tr>
<td>2</td>
<td>0.251</td>
<td>7</td>
<td>0.251</td>
<td>12</td>
<td>0.251</td>
<td>17</td>
<td>0.251</td>
<td>21</td>
<td>0.248</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>7</td>
<td>0.249</td>
<td>12</td>
<td>0.249</td>
<td>17</td>
<td>0.25</td>
<td>22</td>
<td>0.248</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>7</td>
<td>0.249</td>
<td>12</td>
<td>0.249</td>
<td>17</td>
<td>0.25</td>
<td>22</td>
<td>0.248</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>7</td>
<td>0.249</td>
<td>12</td>
<td>0.249</td>
<td>17</td>
<td>0.25</td>
<td>22</td>
<td>0.248</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>8</td>
<td>0.25</td>
<td>13</td>
<td>0.25</td>
<td>18</td>
<td>0.248</td>
<td>23</td>
<td>0.248</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>8</td>
<td>0.25</td>
<td>13</td>
<td>0.25</td>
<td>18</td>
<td>0.248</td>
<td>23</td>
<td>0.248</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>8</td>
<td>0.25</td>
<td>13</td>
<td>0.25</td>
<td>18</td>
<td>0.248</td>
<td>23</td>
<td>0.248</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>8</td>
<td>0.25</td>
<td>13</td>
<td>0.25</td>
<td>18</td>
<td>0.248</td>
<td>23</td>
<td>0.248</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>8</td>
<td>0.249</td>
<td>13</td>
<td>0.25</td>
<td>18</td>
<td>0.249</td>
<td>23</td>
<td>0.249</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>8</td>
<td>0.249</td>
<td>13</td>
<td>0.25</td>
<td>18</td>
<td>0.249</td>
<td>23</td>
<td>0.249</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>8</td>
<td>0.249</td>
<td>13</td>
<td>0.25</td>
<td>18</td>
<td>0.249</td>
<td>23</td>
<td>0.249</td>
</tr>
<tr>
<td>4</td>
<td>0.249</td>
<td>9</td>
<td>0.246</td>
<td>14</td>
<td>0.25</td>
<td>19</td>
<td>0.251</td>
<td>24</td>
<td>0.251</td>
</tr>
<tr>
<td>4</td>
<td>0.249</td>
<td>9</td>
<td>0.246</td>
<td>14</td>
<td>0.25</td>
<td>19</td>
<td>0.251</td>
<td>24</td>
<td>0.251</td>
</tr>
<tr>
<td>4</td>
<td>0.249</td>
<td>9</td>
<td>0.246</td>
<td>14</td>
<td>0.25</td>
<td>19</td>
<td>0.251</td>
<td>24</td>
<td>0.251</td>
</tr>
<tr>
<td>4</td>
<td>0.253</td>
<td>9</td>
<td>0.25</td>
<td>14</td>
<td>0.251</td>
<td>19</td>
<td>0.248</td>
<td>24</td>
<td>0.248</td>
</tr>
<tr>
<td>4</td>
<td>0.254</td>
<td>9</td>
<td>0.248</td>
<td>14</td>
<td>0.254</td>
<td>19</td>
<td>0.25</td>
<td>24</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.254</td>
<td>9</td>
<td>0.248</td>
<td>14</td>
<td>0.254</td>
<td>19</td>
<td>0.25</td>
<td>24</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.254</td>
<td>9</td>
<td>0.248</td>
<td>14</td>
<td>0.254</td>
<td>19</td>
<td>0.25</td>
<td>24</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0.249</td>
<td>9</td>
<td>0.251</td>
<td>14</td>
<td>0.251</td>
<td>19</td>
<td>0.252</td>
<td>24</td>
<td>0.252</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>10</td>
<td>0.25</td>
<td>15</td>
<td>0.252</td>
<td>20</td>
<td>0.254</td>
<td>25</td>
<td>0.254</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>10</td>
<td>0.25</td>
<td>15</td>
<td>0.252</td>
<td>20</td>
<td>0.254</td>
<td>25</td>
<td>0.254</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>10</td>
<td>0.248</td>
<td>15</td>
<td>0.251</td>
<td>20</td>
<td>0.251</td>
<td>25</td>
<td>0.251</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>10</td>
<td>0.248</td>
<td>15</td>
<td>0.251</td>
<td>20</td>
<td>0.251</td>
<td>25</td>
<td>0.251</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>10</td>
<td>0.248</td>
<td>15</td>
<td>0.251</td>
<td>20</td>
<td>0.251</td>
<td>25</td>
<td>0.251</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>10</td>
<td>0.25</td>
<td>15</td>
<td>0.251</td>
<td>20</td>
<td>0.251</td>
<td>25</td>
<td>0.251</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>10</td>
<td>0.25</td>
<td>15</td>
<td>0.251</td>
<td>20</td>
<td>0.251</td>
<td>25</td>
<td>0.251</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>10</td>
<td>0.25</td>
<td>15</td>
<td>0.251</td>
<td>20</td>
<td>0.251</td>
<td>25</td>
<td>0.251</td>
</tr>
</tbody>
</table>

Variable Control Charts – X-Bar-R

- **Xbar/IMR Chart Avg**: 0.2501
- **Range Chart Avg**: 0.0044
- **Rbar/d2**: 0.001892
- **Number of samples**: 100
- **Subgroup size**: 5
- **Number of subgroups**: 20

Upper Limits:
- Xbar one sigma: 0.006
- Xbar two sigma: 0.008
- Xbar three sigma: 0.009
- Xbar one sigma Lower Limit: 0.249
- Xbar two sigma Lower Limit: 0.249
- Xbar three sigma Lower Limit: 0.248

Lower Limits:
- Rbar one sigma: n/a
- Rbar two sigma: 0.003
- Rbar three sigma: 0.001
Variable Control Charts – X-Bar-R

• When to Use

– When you have variable data
– When data are generated frequently
– When you want to detect small changes
– Useful for data that does not form a normal distribution
– Useful manufacturing – sampling to represent a larger population
Variable Control Charts – X-Bar-R

• Several Purposes

  – Used for control
  – Used for analyses
  – Used for education, communication and documentation
Variable Control Charts – X-Bar-R

• Procedure for setting up X-Bar-R control charts

  – **Step 1:**
    – Choose what to measure

  – **Step 2:**
    – Determine the appropriate time period for collecting the data
    – Determine the number of data points per subgroup (n) and the number of subgroups (k) (minimum = 20)
    – Within each subgroup, samples should as alike as possible
Variable Control Charts – X-Bar-R

• Procedure for setting up X-Bar-R control charts
  
  – Step 3:
  – Set up forms for data
Variable Control Charts – X-Bar-R

<table>
<thead>
<tr>
<th>DATE TIME/TURN</th>
<th>SUM</th>
<th>AVERAGE X</th>
<th>RANGE R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Variable Control Charts – X-Bar-R

• Procedure for setting up X-Bar-R control charts

  – **Step 4:**
    – Collect the samples and record the measurements

  – **Step 5:**
    – With raw data, construct a histogram of the individual data points and the averages of the subgroups
      – Check for normality – Anderson-Darling Test
Variable Control Charts – X-Bar-R

• Histogram of Individual Data Points
Variable Control Charts – X-Bar-R

- Histogram of the Average of the Subgroups
Variable Control Charts – X-Bar-R

- Normality Test for Individual Data Points
Variable Control Charts – X-Bar-R

- Normality Test for the Average of the Subgroups
Variable Control Charts – X-Bar-R

- Procedure for setting up X-Bar-R control charts
- Calculate various statistics and determine the control limits for charts
  - Step 6:
    - Calculate the averages $\bar{x}$ (X-bar)
  - Step 7:
    - Calculate the average of the averages $\bar{\bar{x}}$ (X-double-bar)
Variable Control Charts – X-Bar-R

- Procedure for setting up X-Bar-R control charts
  - Calculate various statistics and determine the control limits
    - **Step 8:**
      - Determine the range for the samples
    - **Step 9:**
      - Calculate the average of the ranges $\bar{R}$
    - **Step 10:**
      - Calculate the control limits for the range and $\bar{x}$ (X-bar) charts
Variable Control Charts – X-Bar-R

<table>
<thead>
<tr>
<th>DATE</th>
<th>1/10</th>
<th>1/11</th>
<th>1/12</th>
<th>1/13</th>
<th>1/14</th>
<th>1/15</th>
<th>1/16</th>
<th>1/17</th>
<th>1/18</th>
<th>1/19</th>
<th>1/20</th>
<th>1/21</th>
<th>1/22</th>
<th>1/23</th>
<th>1/24</th>
<th>1/25</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIME/TURN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.44</td>
<td>2.51</td>
<td>2.49</td>
<td>2.45</td>
<td>2.45</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
</tr>
<tr>
<td>2</td>
<td>2.51</td>
<td>2.46</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
</tr>
<tr>
<td>3</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
</tr>
<tr>
<td>4</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
</tr>
<tr>
<td>5</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.45</td>
</tr>
<tr>
<td>SUM</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
<td>12.9</td>
</tr>
<tr>
<td>AVERAGE R</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
<td>2.54</td>
</tr>
<tr>
<td>RANGE R</td>
<td>0.003</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
</tbody>
</table>
### Average and Range Control Chart

**Inland Steel Quality System**

**Variable Control Charts – X-Bar-R**

<table>
<thead>
<tr>
<th>DATE</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
<th>1/10</th>
</tr>
</thead>
</table>

| 1 | 249 | 251 | 250 | 250 | 247 | 250 | 246 | 250 | 250 | 245 | 250 | 250 | 250 | 249 | 250 | 250 | 250 | 250 | 250 |
| 2 | 251 | 246 | 250 | 250 | 244 | 250 | 250 | 250 | 250 | 250 | 250 | 248 | 250 | 250 | 250 | 250 | 250 | 250 | 250 |
| 3 | 251 | 252 | 246 | 250 | 250 | 250 | 249 | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 | 250 |

| AVERAGE \( \bar{X} \) | 2498 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 | 2500 |
| RANGE \( R \) | .003 | .006 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 | .005 |
Variable Control Charts – X-Bar-R

Process: Pin Diameter
Data dates: Calculation date:

Step 1. Calculate average $\bar{X}$ and range $R$ (the difference between the highest and lowest values) for each subgroup. Record on chart.
Number of values in each subgroup $n = 5$
Number of subgroups to be used $k = 20$

Step 2. Look up control limit factors.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$A_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.880</td>
<td>3.267</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.023</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.729</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.577</td>
<td></td>
<td>$A_2 = 0.577$</td>
</tr>
<tr>
<td>6</td>
<td>0.462</td>
<td></td>
<td>$D_3 = 0$</td>
</tr>
<tr>
<td>7</td>
<td>0.419</td>
<td>0.076</td>
<td>$D_4 = 2.114$</td>
</tr>
</tbody>
</table>

$D_3 = N/A$

Step 3. Calculate averages ($\bar{X}$ and $\bar{R}$).

Sum of the averages $\sum X = \frac{\sum X}{k} = \frac{5.0014}{20} = 0.25007$
Average of the averages $\bar{X} = \frac{\sum X}{k} = \frac{5.0014}{20} = 0.25007$
Avg of the ranges $\bar{R} = \frac{\sum R}{k} = \frac{0.045}{20} = 0.0045$


$3\hat{\sigma}$ estimate for $\bar{X}$ chart:
$\hat{\sigma} = \frac{\sum R}{k\bar{R}} = \frac{0.045}{20 \times 0.0045} = 0.0026$
Upper control limit for $\bar{X}$ chart $UCL_\bar{X} = \bar{X} + 3\hat{\sigma} = 0.25007 + 0.0026 = 0.25267 = 0.253$
Lower control limit for $\bar{X}$ chart $LCL_\bar{X} = \bar{X} - 3\hat{\sigma} = 0.25007 - 0.0026 = 0.24747 = 0.2475$

Upper control limit for $R$ chart $UCL_R = D_4 \times \bar{R} = 2.114 \times 0.0045 = 0.00495$
Lower control limit for $R$ chart $LCL_R = D_3 \times \bar{R} = N/A \times 0.0045 = 0.0045$
Variable Control Charts – X-Bar-R

Step 1. Calculate average $\bar{X}$ and range $R$ (the difference between the highest and lowest values) for each subgroup. Record on chart.

Number of values in each subgroup = $n = 5$

Number of subgroups to be used = $k = 20$

Step 2. Look up control limit factors.

$$\begin{array}{ccc}
 n & A_2 & D_3 & D_4 \\
 2 & 1.880 & - & 3.267 \\
 3 & 1.023 & - & 2.574 \\
 4 & 0.729 & - & 2.282 \\
 5 & \frac{0.577}{10.577} & - & 2.114 \\
 6 & 0.483 & - & 2.004 \\
 7 & 0.419 & 0.076 & 1.924 \\
\end{array}$$

$$A_2 = \frac{0.577}{10.577}$$

$$D_3 = N/A$$

$$D_4 = 2.114$$

Step 3. Calculate averages ($\bar{X}$ and $\bar{R}$).

Sum of the averages = $\Sigma \bar{X}$

Average of the averages = $\bar{X} = \frac{\Sigma \bar{X}}{k} = \frac{5.0014}{20} = 0.25007$

Sum of the ranges = $\Sigma R$

Average of the ranges = $\bar{R} = \frac{\Sigma R}{k} = \frac{0.9}{20} = 0.045$
Variable Control Charts – X-Bar-R


\[ 3\sigma \text{ estimate for } \bar{X} \text{ chart} = 3\hat{\sigma}_{\bar{X}} = A_2 \times \bar{R} \]

\[ = \frac{\bar{R}}{\bar{X}} \times \frac{0.045}{25007} = 0.0026 \]

Upper control limit for \( \bar{X} \) chart

\[ UCL_{\bar{X}} = \bar{X} + 3\hat{\sigma}_{\bar{X}} \]

\[ = 25007 + 0.0026 = 25267.2527 \]

Lower control limit for \( \bar{X} \) chart

\[ LCL_{\bar{X}} = \bar{X} - 3\hat{\sigma}_{\bar{X}} \]

\[ = 25007 - 0.0026 = 24747.2475 \]

Upper control limit for \( R \) chart

\[ UCL_R = D_4 \times \bar{R} \]

\[ = 2.114 \times \frac{0.045}{25007} = 0.0095 \]

Lower control limit for \( R \) chart

\[ LCL_R = D_3 \times \bar{R} \]

\[ = \frac{N/A}{\bar{X}} = 0 \]
Variable Control Charts – X-Bar - R

- Procedure for setting up X-Bar-R control charts
  - Step 11:
    - Determine the scale for the plots
  - Step 12:
    - Draw the control limits and averages and plot the data for both charts
    - Connect the dots
Variable Control Charts – X-Bar - R
Variable Control Charts –X-Bar -R

X-Bar Chart - Pin Diameter

Range Chart - Pin Diameters
Variable Control Charts – X-Bar-R

• Analysis
  – Step 1:
    – Check the range chart for out-of-control points
    – If out-of-control, investigate reasons
    – Cannot proceed to Step 2 until out-of-control reasons explained and/or range chart back in control
      – Recalculate control limits without out-of-control points
  – Step 2:
    – Check the x-bar chart for out-of-control signals
    – If out-of-control, investigate reasons
Variable Control Charts – X-Bar-R

X-Bar of Undercut Diameter

Range of Undercut Diameter

- Three Sigma Limit
- Two Sigma Limit
- One Sigma Limit
- Average
- A single point outside the control limits
- Two of three pts outside the two sigma limit
- Four of Five pts outside the one sigma limit
- Eight in a row on the same side of centerline
Variable Control Charts – X-Bar-R

X-Bar of Undercut Diameter

Range of Undercut Diameter

Three Sigma Limit
Two Sigma Limit
One Sigma Limit
Average

A single point outside the control limits
Two of three pts outside the two sigma limit
Four of Five pts outside the one sigma limit
Eight in a row on the same side of centerline
Variable Control Charts – X-Bar-R

- Out-of-Control Signals
  - Any points outside of control limits
  - 2 out of 3 successive points are on the same side of the centerline and further than $2\sigma$ from it
  - 4 out of 5 successive points are on the same side of the centerline and further than $1\sigma$ from it.
  - A run of eight in a row on the same side of the control limits
  - 6 successive points increasing or decreasing
  - Consistent or persistent patterns
  - Other
  - Limit rules for out-of-control conditions in SPC control charts
Variable Control Charts – X-Bar-R

• **Out-of-Control Signals**
  – Considerations
    • Signal rules are based on statistics and a normal and predictable curve
    • Signals do not indicate whether patterns are undesirable or desirable
    • Data points cannot show autocorrelation – successful points related to the preceding points
      – Data collection time period important
      – Autocorrelation test
    • Control limits are not specification limits
    • Control limits are only recalculated when there is a permanent change in the process
Variable Control Charts – X-Bar-R

• Out-of-Control Signals
  – Considerations (continued)
    • Most useful data are plotted as soon as they are generated by the people working the process
    • Software is available
    • Control charts are often applied incorrectly
Control Chart Decision Tree

Choose Appropriate Control Chart

Attribute Data
Counted & plotted as discrete events

Defect Data
Constant sample size
- c Chart
Variable sample size
- u Chart

Defective Data
Constant sample size ≥ 50
- np Chart
Variable sample size ≥ 50
- p Chart

Continuous Data
Measured & plotted on a continuous scale

Sample size = 1
- I and MR
Sample is large, usually ≥ 10
- X-Bar and s
Sample is small, usually 3 to 5
- X-Bar and R
Variable Control Charts – X-Bar-S

• Similar to X-Bar-R chart except use standard deviation in place of range

• When to use
  – Variable data
  – Need lots of data, n ≥ 10
  – Need to detect very small changes

• Standard deviation statistic calculations
Variable Control Charts – Individual – Moving Range

- Study variable data that are not generated frequently enough for an X-Bar-R Chart

- When to use
  - Variable data
  - Normal distribution
  - Cannot use X-Bar-R chart due to infrequent data
  - Cannot use X-Bar-R chart because the measurement remains constant for a relatively long period of time before process changes

- Plot individual values and the moving range – difference between successive individual values
Variable Control Charts – Individual – Moving Range

Process: ______________________  Calculated by: ______________________
Data dates: ___________________ Calculation date: ___________________

Step 1. Calculate $\bar{X}$.
Number of values = $k = _____$
Sum of the values = $\sum X = _____$
Average = $\bar{X} = \frac{\sum X}{k}$

Step 2. Calculate $\overline{MR}$.
Calculate the ranges (the difference between two consecutive data points) and record on the form. Ignore negative signs.
Number of moving ranges = $k - 1 = _____$
Sum of the moving ranges = $\sum MR = _____$
Average moving range = $\overline{MR} = \frac{\sum MR}{k - 1}$

Step 3. Calculate control limits.
Estimate 3 standard deviations = $3\hat{\sigma}_x = 2.66 \times \overline{MR}$

Upper control limit for $X$ chart = $UCL_X = \bar{X} + 3\hat{\sigma}_x$
Lower control limit for $X$ chart = $LCL_X = \bar{X} - 3\hat{\sigma}_x$
Upper control limit for $MR$ chart = $UCL_{MR} = 3.267 \times \overline{MR}$
Variable Control Charts – Other

• Moving Average-Moving Range
  • Each successive subgroup drops the oldest measurement from the previous subgroup
  • Cannot use X-Bar-R - measurements remain constant
  • Cannot use I-MR – data are not normal

• Target Chart
  • Allows the same characteristic from different parts or products to be plotted on the same chart

• Median and Range
  • Good to use when data are normal and are not very often disturbed by assignable causes
Control Chart Decision Tree

Choose Appropriate Control Chart

Attribute Data
Counted & plotted as discrete events

Defect Data
Constant sample size
- c Chart
Variable sample size
- u Chart

Defective Data
Constant sample size ≥ 50
- np Chart
Variable sample size ≥ 50
- p Chart

Continuous Data
Measured & plotted on a continuous scale

Sample size = 1
- I and MR
Sample is large, usually ≥ 10
- X-Bar and s
Sample is small, usually 3 to 5
- X-Bar and R
Attribute Control Charts

- Used for go-no go, defects, counts

- Use when you need to monitor a non-measurable in your product or process

- Only one control chart

- Use variable data where possible
Attribute Control Charts – P/NP-Charts

- Used for go-no go, defects, counts

- P-Chart
  - Used to study proportion of non-conforming or defective items
  - Items are either good or bad
  - P equals the number of defective pieces divided by sample size
  - Sample size can vary
  - Sample size should be approximately greater than 50
Attribute Control Charts – P/NP-Charts

- NP-Chart
  - Same as P-Chart but sample size is constant
  - Can also use P-Chart if sample size is constant
  - Plot number instead of percent defective
• C-Chart
  • Count chart used to study the number of non-conformities or defects

• When to use:
  • Counting non-conformities
  • Each sample must have the same opportunity for non-conformities to occur
  • More than one non-conformity can be counted per item or per area
  • Sample size (length, area, etc.) remains constant
Attribute Control Charts – C/U- Charts

• U-Chart
  • Similar to C-Chart but used to study the proportion of non-conformities

• When to use:
  • Counting non-conformities and
    • Sample size varies or
    • Where the opportunity for non-conformities changes from one sample to the next
Attribute Control Charts – P-Chart

<table>
<thead>
<tr>
<th>Date</th>
<th>Time/ Turn</th>
<th>Successes / Failures</th>
<th>n</th>
<th>p, np, u, c</th>
<th>3σ</th>
<th>UCL</th>
<th>LCL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample Frequency</th>
<th>Unit of Measurement</th>
<th>Type of Measurement Equipment</th>
<th>Operator</th>
<th>Chart No.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Inland Steel Quality System

51
Attribute Control Charts – P-Chart

P Chart of Defective Panels

- Proportion
- Sample

UCL = 0.7372
\( \bar{p} = 0.5208 \)
LCL = 0.3045
### Attribute Control Charts – P-Chart

1. **CONTROL LIMITS FOR \( \bar{p} \):**
   \[
   \bar{p} = \frac{\sum np}{\Sigma n} = \frac{\Sigma np}{\Sigma n} =
   \]
   \[
   \sigma_{\bar{p}} = \sqrt{\bar{p}(1 - \bar{p})} = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} =
   \]
   \[
   UCL_{\bar{p}} = \bar{p} + 3\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} =
   \]
   \[
   LCL_{\bar{p}} = \bar{p} - 3\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} =
   \]

2. **CONTROL LIMITS FOR \( np \):**
   \[
   np = \frac{\Sigma np}{K} = np = \frac{np}{n} =
   \]
   \[
   \sigma_{np} = \sqrt{np(1 - \bar{p})} =
   \]
   \[
   UCL_{np} = np + 3\sqrt{np(1 - \bar{p})} =
   \]
   \[
   LCL_{np} = np - 3\sqrt{np(1 - \bar{p})} =
   \]

3. **CONTROL LIMITS FOR \( \bar{c} \):**
   \[
   \bar{c} = \frac{\Sigma c}{K} =
   \]
   \[
   \sigma_{\bar{c}} = \sqrt{\frac{\Sigma c}{K}} =
   \]
   \[
   UCL_{\bar{c}} = \bar{c} + 3\sqrt{\frac{\Sigma c}{K}} =
   \]
   \[
   LCL_{\bar{c}} = \bar{c} - 3\sqrt{\frac{\Sigma c}{K}} =
   \]

4. **CONTROL LIMITS FOR \( \bar{u} \):**
   \[
   \bar{u} = \frac{\Sigma u}{\Sigma n} =
   \]
   \[
   \sigma_{\bar{u}} = \sqrt{\frac{\Sigma u}{\Sigma n}} =
   \]
   \[
   UCL_{\bar{u}} = \bar{u} + 3\sqrt{\frac{\Sigma u}{\Sigma n}} =
   \]
   \[
   LCL_{\bar{u}} = \bar{u} - 3\sqrt{\frac{\Sigma u}{\Sigma n}} =
   \]
Summary

• Control Charts can used for several purposes
  – Monitor process variables and parameters - SPC
  – Other continuous improvement and/or problem solving activities

• Variable and Attribute Control Charts – try to use variable

• In SPC, important for operators to fill out and react to out of control conditions

• Understand random and special cause variation and stability

• Your process or engineering knowledge will solve the problems!!!!!
Computer Programs for Control Charts

• Minitab
  – Good for statistical calculations and charts

• SAS Jump
  – Similar to Minitab

• Excel Templates
  – ASQ for X-Bar-Range

• Many offered online

• Tailor your own
Bibliography and Good References


